# The Project for Human Resource Development Scholarship by Japanese Grant Aid (JDS)

## Basic Mathematics Aptitude Test
## 2020

Note:
- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are 'Basic Math,' and Part III and Part IV are 'Applied Math.'

*Name* : _____

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

1.  $(-2) \times (2 - 3) \times (-1) + 3$

Answer : _____

2.  $\frac{1}{3} \times \left( \frac{1}{5} \div \frac{1}{10} - \frac{2}{3} \right) - \frac{1}{3}$

Answer : _____

3.  $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

Answer : _____

4.  $\left( \left( \frac{1}{2} \right)^2 \times \left( \frac{1}{4} \right)^{0.5} \right)^{-4} \div 8^2$

Answer : _____

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equation for y.

$$2y = \frac{3y - 2}{2}$$

Answer : _____

2. Solve the following simultaneous equations for a and b.
$$a + 3b = 5$$
$$-2a + 3b = 8$$

Answer : _____

3. Find the region of x satisfying the following inequality.

$$|x - 1| < 3$$

Answer : _____

4. Consider the straight line in the (x,y)-plane that passes through the point (0,a). Assume that the slope is $-2$ and the x-intercept is (2,0). Find the value of a.

Answer : _____

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Find the region of x satisfying the following inequality.

   $2x^2 < 5x - 3$

   Answer : _____

2. Solve the following equation for x.

   $2\log_{10}(x) = \log_{10}(x + 6)$

   Answer : _____

3. Find the positive integer x satisfying the following equality.

   $$\sum_{k=1}^{x} k = 55$$

   Answer : _____

4. Consider the following five values,
   $\{-2, \ 9, \ 20, \ 3, \ 15\}$.
   Suppose that the average of these five values is $3^{4x}$. Find the value of x.

   Answer : _____

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1.  Determine the first-order derivative of the following. Note that e is a mathematical constant which is the base of the natural logarithm.

$$y = x^3 e^x - 5$$

Answer :  _____

2.  Solve the following equation for x. Assume $x > 0$. Note that e is a mathematical constant which is the base of the natural logarithm.

$$\int_1^{3x} \frac{1}{z} dz = \log_e 5$$

Answer :  _____

3.  Let $A = \begin{bmatrix} a & -1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$. Assume that the determinant of A is 1. Find $A^{-1}B$.

Answer : _____

4.  Consider the profit function, $\pi(q) = (100 - 2q)q - \frac{1}{2}q^2$, where q is the output, and $\pi(q)$ is the profit with output q as given. Find the optimal level of output maximizing the profit.

Answer : _____

**[PART V]  Fill in the following blanks with correct answers.**

1.  Find the first derivative of the following.
      $f(x) = \sin(3 - x)$.

Answer :  _____

2.  Consider a sequence $\{a_k\}_{k=1}^{\infty}$ and its series
$$S_n = \sum_{k=1}^{n} a_k = (n + 1)^2$$
for $n = 1,2,3, \cdots$. Find the value of $a_{10}$.

Answer :  _____

3. Consider two vectors, $\vec{a} = (1, 1)$ and $\vec{b} = (-1, 0)$. Let $\theta$ denote the angle between $\vec{a}$ and $\vec{b}$. Find the value of $\cos\theta$.

Answer :

4. A football team consisting of 6 boys and 5 girls will be formed from a group of 7 boys and 7 girls. Find how many different teams can be formed from the group.

Answer :