The Project for Human Resource Development Scholarship by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test 2020

Solution

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are ‘Basic Math,’ and Part III and Part IV are ‘Applied Math.’

Name : _____

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

1. $(-2) \times (2-3) \times (-1) + 3$

$= (-2) + 3 = 1$

Answer : 1

2. $\frac{1}{3} \times \left(\frac{1}{5} \div \frac{1}{10} - \frac{2}{3}\right) - \frac{1}{3}$

$= \frac{1}{3} \times \left(\frac{1}{5} \times \frac{10}{1} - \frac{2}{3}\right) - \frac{1}{3} = \frac{1}{3} \times \left(2 - \frac{2}{3}\right) - \frac{1}{3} = \frac{1}{3} \times \frac{4}{3} - \frac{1}{3} = \frac{4}{9} - \frac{1}{3} = \frac{1}{9}$

Answer : $\frac{1}{9}$

3. $\left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right)$

$= 5 - 3 = 2$

Answer : 2

4. $\left(\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{4}\right)^{0.5}\right)^{-4} \div 8^2$

$= (2^{-2} \times 2^{-1})^{-4} \div 2^6 = 2^{12} \times 2^{-6} = 2^6 = 64$

Answer : 64

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equation for y.

$$2y = \frac{3y - 2}{2}$$

$4y = 3y - 2 \rightarrow y = -2$

Answer : $y = -2$

2. Solve the following simultaneous equations for a and b.

$$a + 3b = 5$$
$$-2a + 3b = 8$$

$a = -1,\ b = 2$

Answer : $a = -1, b = 2$

3. Find the region of x satisfying the following inequality.

$$|x - 1| < 3$$

$-2 < x < 4$

Answer : $-2 < x < 4$

4. Consider the straight line in the (x,y)-plane that passes through the point (0,a). Assume that the slope is -2 and the x-intercept is $(2,0)$. Find the value of a.

The slope is -2, and the y-intercept is a. Thus, we can write $y = -2x + a$. Since the x-intercept is 2, we have $a = 4$.

Answer : 4

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Find the region of x satisfying the following inequality.

$2x^2 < 5x - 3$

$(2x-3)(x-1) < 0 \rightarrow 1 < x < \frac{3}{2}$

Answer : $1 < x < \frac{3}{2}$

2. Solve the following equation for x.

$2\log_{10}(x) = \log_{10}(x+6)$

$x^2 - x - 6 = (x+2)(x-3) = 0 \rightarrow x = -2, 3$
Since $x > 0$, we have $x = 3$

Answer : $x = 3$

3. Find the positive integer x satisfying the following equality.

$$\sum_{k=1}^{x} k = 55$$

$\sum_{k=1}^{x} k = \frac{x(x+1)}{2} = 55 \rightarrow x(x+1) = 110 \rightarrow x^2 + x - 110 = (x-10)(x+11) = 0$
$\rightarrow x = 10, -11$

Answer : 10

4. Consider the following five values,
$\{-2,\ 9,\ 20,\ 3,\ 15\}$.

Suppose that the average of these five values is 3^{4x}. Find the value of x.

$\frac{-2+9+20+3+15}{5} = 9 = 3^2 = 3^{4x} \rightarrow 4x = 2$

Answer : $x = \frac{1}{2}$

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1. Determine the first-order derivative of the following. Note that e is a mathematical constant which is the base of the natural logarithm.

   $y = x^3e^x - 5$

$y' = 3x^2e^x + x^3e^x$

Answer : $y' = 3x^2e^x + x^3e^x$

2. Solve the following equation for x. Assume $x > 0$. Note that e is a mathematical constant which is the base of the natural logarithm.

   $\int_1^{3x} \frac{1}{z} dz = \log_e 5$

$\int_1^{3x} \frac{1}{z} dz = \log_e(3x) - \log_e(1) = \log_e(3x) = \log_e(5)$. Thus, $3x = 5 \rightarrow x = 5/3$

Answer : $x = \frac{5}{3}$

3. Let $A = \begin{bmatrix} a & -1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$. Assume that the determinant of A is 1. Find $A^{-1}B$.

$\det(A) = a + 2 = 1 \rightarrow a = -1$
$A^{-1} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \rightarrow A^{-1}B = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}$

Answer : $\begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}$

4. Consider the profit function, $\pi(q) = (100 - 2q)q - \frac{1}{2}q^2$, where q is the output, and $\pi(q)$ is the profit with output q as given. Find the optimal level of output maximizing the profit.

   $\pi'(q) = 100 - 4q - q$. The first-order condition is: $100 - 5q = 0 \rightarrow q = 20$.

Answer : $q = 20$

[PART V] Fill in the following blanks with correct answers.

1. Find the first derivative of the following.
   $f(x) = \sin(3 - x)$.

Solution
$f'(x) = \cos(3 - x) \times (-1) = -\cos(3 - x)$.

Answer : $-\cos(3 - x)$

2. Consider a sequence $\{a_k\}_{k=1}^{\infty}$ and its series
   $S_n = \sum_{k=1}^{n} a_k = (n + 1)^2$
   for $n = 1,2,3,\cdots$. Find the value of a_{10}.

Solution
For $k = 2,3,4,\cdots$, we have that $a_k = S_k - S_{k-1} = (k + 1)^2 - k^2 = 2k + 1$. For $k = 1$, $a_1 = S_1 = 4$. Thus, $a_k = 2k + 1$ and $a_{10} = 21$.

Answer : 21

3. Consider two vectors, $\vec{a} = (1, 1)$ and $\vec{b} = (-1, 0)$. Let θ denote the angle between $\vec{a}$ and $\vec{b}$. Find the value of $\cos\theta$.

Solution

The inner product of $\vec{a} = (1,1)$ and $\vec{b} = (-1,0)$ is $\vec{a} \cdot \vec{b} = 1 \times (-1) + 1 \times 0 = -1$.
Since $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = 1$, and $\vec{a} \cdot \vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$, we obtain:
$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Answer : $\cos\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

4. A football team consisting of 6 boys and 5 girls will be formed from a group of 7 boys and 7 girls. Find how many different teams can be formed from the group.

Solution

${}_7C_6 \times {}_7C_5 = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{7 \times 6 \times 5 \times 4 \times 3}{5 \times 4 \times 3 \times 2 \times 1} = 7 \times \frac{7 \times 6}{2} = 147.$

Answer : 147