The Project for Human Resource Development Scholarship by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test 2021

Solution

Note:

- The test is a computer-scored multiple-choice test.
- You have 60 minutes to complete.
- No calculators are allowed.
- Part I and II are ‘Basic Math,’ and Part III, IV and V are ‘Applied Math.’
- Select one(1) integer 0 to 9 for each square.
- Each square correspond to each answer number of computer-scored answer sheet.

Name: ______________________________

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

➢ $(-3) \times (1-3) \times (12-3)$

$= (-3) \times (-2) \times 9 = 54$

Answer : 54

➢ $\left(\frac{1}{2} \div \frac{1}{3} - \frac{2}{3}\right) \times \left(\frac{1}{2} \div \frac{1}{3} + \frac{2}{3}\right)$

$= \left(\frac{3}{2} - \frac{2}{3}\right) \times \left(\frac{3}{2} + \frac{2}{3}\right) = \frac{9}{4} - \frac{4}{9} = \frac{81-16}{36} = \frac{65}{36}$

Answer : $\frac{65}{36}$

➢ $\left(\sqrt{5} + 2\right)^2$

$= 5 + 4\sqrt{5} + 4 = 9 + 4\sqrt{5}$

Answer : $9 + 4\sqrt{5}$

➢ $\left(\left(\frac{1}{2}\right)^{-2.5} \times \left(\frac{1}{4}\right)^{0.25}\right)^{-4}$

$= (2^{2.5} \times 2^{-0.5})^{-4} = (2^2)^{-4} = 2^{-8} = \frac{1}{256}$

Answer : $\frac{1}{256}$

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

- Solve the following equation for x.

$$2 = \frac{5x - 1}{x + 2}$$

$2x + 4 = 5x - 1 \rightarrow 3x = 5 \rightarrow x = \frac{5}{3}$

Answer : $x = \frac{5}{3}$

- Solve the following simultaneous equations for a and b.

$a + b = 16$
$ab = 64$

$a = 8,\ b = 8$

Answer : $a = 8, b = 8$

- Find the region of x satisfying the following inequality.

$|x| \leq x^2$

$x < 0 \rightarrow -x \leq x^2 \rightarrow x \leq -1$
$x \geq 0 \rightarrow x \leq x^2 \rightarrow 1 \leq x$

Answer : $x \leq -1, 1 \leq x$

- Consider the straight line in the (x,y)-plane that passes through the point (a+1, a). Assume that the slope is -1 and the x-intercept is $(5,0)$. Find the value of a.

The slope is -1, and the x-intercept is 5. Thus, we can write $y = -x + 5$. Since the line passes through (a+1, a), we have $a = 2$.

Answer : 2

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

➢ Find the region of x satisfying the following inequality.

$2^{x^2} < 2^{64}$

$x^2 < 64 \rightarrow -8 < x < 8$

Answer : $-8 < x < 8$

➢ Solve the following equation for x.

$\log_{10}(x) - \log_{10}\left(\frac{1}{x}\right) = \log_{10}(10 - 3x)$

$x^2 + 3x - 10 = (x + 5)(x - 2) = 0 \rightarrow x = -5,2$
Since $x > 0$, we have $x = 2$

Answer : $x = 2$

➢ Consider a sequence series $\{x_k\}_{k=1}^{\infty}$ with $x_k = 2k - 1$. Consider the series $S_n = \sum_{k=1}^{n} x_k$. Find the smallest integer of n satisfying $S_n > 120$.

$S_n = 2 \times \frac{n(n+1)}{2} - n = n^2 > 120 \rightarrow$ the smallest $n = 11$

Answer : 11

➢ Consider the following five values,
$\{-2,\ 5,\ -1,\ 3, -\ 5\}$.

Let x and y be the average and median of these five values, respectively. Find the value of $\log_{10}(x - y)$.

$x = 0, y = -1 \rightarrow x - y = 1 \rightarrow \log_{10}(x - y) = 0$

Answer : 0

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

- Determine the second-order derivative of the following. Assume $x > 0$. Note that e is a mathematical constant which is the base of the natural logarithm.

$$y = \int_0^x (2z)dz - \log_e(x^3)$$

$y' = 2x - \frac{3}{x}$ → $y'' = 2 + \frac{3}{x^2}$

Answer : $y'' = 2 + \frac{3}{x^2}$

- Assume that $b > 1$. Find the following value.

$$\lim_{n\to\infty} \frac{2b^n}{10 + 3b^n}$$

$\frac{2b^n}{10+3b^n} = \frac{2}{10/b^n+3} \to \frac{2}{3}$.

Answer : $\frac{2}{3}$

- Let $A = \begin{bmatrix} 1 & 1 \\ -2 & a \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. Assume that the determinant of A is 2. Find $A^{-1}B$.

$\det(A) = a + 2 = 2$ → $a = 0$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$ → $A^{-1}B = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$

Answer : $\begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$

- Find the values of x and y that solve the following constrained maximization problem:
  Maximize $\sqrt{xy}$ subject to $x + y = 10$.

  $y = 10 - x$ → $\sqrt{xy} = \sqrt{x(10 - x)} = \sqrt{10x - x^2}$ → $10 - 2x = 0$ → $x = 5$.

Answer : $x = 5$, $y = 5$

[PART V] Fill in the following blanks with correct answers.

- Find the first derivative of the following.

  $f(x) = \sin(x^2)$.

Solution

$f'(x) = \cos(x^2) \times (2x) = 2x\cos(x^2)$.

Answer : $2x\cos(x^2)$

- A continuous random variable follows the following probability density function f. Find the value of a positive constant b.

$$f(x) = \begin{cases} b & \text{if } 0 \le x \le 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Solution

For f to be a probability density function, it must hold that $\int_{-\infty}^{\infty} f(x)dx = 1$.

$\int_{-\infty}^{\infty} f(x)dx = \int_{0}^{0.5} b\,dx = b \times 0.5 = 1 \rightarrow b = 2$

Answer : $b = 2$

➢ Suppose that $\vec{a} = (2x, -1)$ and $\vec{b} = (x, 32)$ are vertical. Find the value of x.

Solution

The inner product of $\vec{a}$ and $\vec{b}$ must be zero.

$\vec{a} \cdot \vec{b} = 2x \times x - 1 \times 32 = 0 \rightarrow x^2 = 16 \rightarrow x = -4, 4$

Answer : $x = -4, 4$

➢ A baseball team consisting of 5 boys and 4 girls will be formed from a group of 6 boys and 7 girls. Find how many different teams can be formed from the group.

Solution

${}_6C_5 \times {}_7C_4 = \frac{6\times5\times4\times3\times2}{5\times4\times3\times2\times1} \times \frac{7\times6\times5\times4}{4\times3\times2\times1} = 6 \times \frac{7\times6\times5}{3\times2\times1} = 6 \times (7 \times 5) = 210.$

Answer : 210
