The Project for Human Resource Development Scholarship by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test 2022

Solution

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are 'Basic Math,' and Part III and Part IV are 'Applied Math.'

Name : ______________________________

【PART I】Calculate the followings.

1. $2 - (2 - 2\times (4 + (2 - 6)))$

$= 2 - (2 - 2\times (4+ (-4)))= 2 - (2 - 2\times 0) =2 - 2 = 0$

Answer： 0

2. $\left(1 + \frac{1}{3} \times \frac{3}{4} \div \frac{1}{4}\right) - \frac{2}{5} \times \frac{10}{4}$

$= \left(1 + \frac{1}{3} \times \frac{3}{4} \times \frac{4}{1}\right) - \frac{1}{1} \times \frac{2}{2} = (1 + 1) - 1 = 1$

Answer： 1

3. $(\sqrt{3} - \sqrt{7}) \times (\sqrt{3} + \sqrt{7})$

$= 3 - 7 = -4$

Answer： -4

4. $\left(2^{-2} \times \left(\frac{1}{2}\right)^{-2}\right)^{-4} \div \left(\frac{1}{3}\right)^{2}$

$=\left(\left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{-2}\right)^{-4} \times \left(\frac{1}{3}\right)^{-2}=\left(\left(\frac{1}{2}\right)^{2-2}\right)^{-4} \times 3^2 = 1 \times 3^2 = 9$

Answer： 9

【PART II】 Answer the following questions.

1. Solve the following equation for x.

$$\left(\frac{10-x}{3}\right)=3x$$

$10 - x = 9x \rightarrow 10 = 10x \rightarrow x = 1$

Answer : $x=1$

2. Solve the following simultaneous equations for x and y.

$$-x+6\mathrm{y}=19$$
$$-x+2\mathrm{y}=7$$

Answer : $x=-1, \quad y=3$

3. Find the region x satisfying the following inequality, where | | indicates the absolute value.

$$|x+3|<2$$
$$-2<(x+3)<2 \rightarrow -2-3<x<2-3 \rightarrow -5<x<-1$$

Answer : $-5 < x < -1$

4. Solve the following.

$$\sum_{n=1}^{5}(2n-1)$$
$$\sum_{n=1}^{5}(2n-1)=2\times\frac{5(5+1)}{2}-5=30-5=25$$

Answer : 25

[PART III] Answer the following questions.

1. Solve the following equation for x.

$$\frac{x^2}{4} = 4$$

$x^2 = 16 \rightarrow x^2 = (\pm 4)^2 \rightarrow x = 4, -4$

Answer : $x = 4, -4$

2. Find the region of x satisfying the following inequality.

$x^2 < 4x - 3$

$x^2 - 4x + 3 < 0 \rightarrow (x-3)(x-1) < 0 \rightarrow 1 < x < 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{4^2 - 4 \times 1 \times 3}}{2 \times 1} = \frac{4 \pm \sqrt{16-12}}{2 \times 1} = 1, 3 \rightarrow 1 < x < 3$

Answer : $1 < x < 3$

3. Solve the following equation for x.

$log_{10}(x) = log_{10}(2x-4)$

$log_{10}(x) = log_{10}(2x-4) \rightarrow x = 2x - 4 \rightarrow -x = -4 \rightarrow x = 4$

Answer : $x = 4$

4. Consider the following five values, $\{1, 2, 7, 6, 4\}$. Suppose that the average of these five values is $log_2(x)$. Find the value of x.

$\frac{(1+2+7+6+4)}{5} = 4 = log_2(x) \rightarrow log_2(x) = 4 \rightarrow x = 2^4 = 16$

Answer : 16

【PART IV】 Answer the following questions.

1. Determine the first-order derivative of the following. Note that e is a mathematical constant which is the base of the natural logarithm.

$$y = 2x^2 + e^x + log_e x + 5$$

Answer： $y' = 4x + e^x + \frac{1}{x}$

2. Find the following definite integral. $\int_{-1}^{0} 2xdx$

$$\int_{-1}^{0} 2xdx \;=\; 2 \times \frac{x^2}{2}\Big]_{-1}^{0} = 0 - \;1 \;= -1$$

Answer： -1

3. Let A=$\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$. Find the inverse matrix of A.

A=$\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ → $\det(A) = 2$ where det(A)is the determinant of the matrix A.

$$A^{-1} = \frac{1}{4 \times 1 - 2 \times 1}\begin{bmatrix} 4 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -0.5 & 0.5 \end{bmatrix}$$

Answer： $\begin{bmatrix} 2 & -1 \\ -0.5 & 0.5 \end{bmatrix}$

4. The profit π is described by the following function: $\pi(q) = (200 - 2q)q - 0.5q^2$, where q is output. Find the output q at which the profit is maximized.

Solution: the first-odder condition is : $\pi'^{(q)} = 200 - 4q - q = 0$ → q=40

Answer： $q = 40$

【PART V】 **Answer the following questions.**

1. Find the first derivative of the following. $f(\theta) = (\sin\theta)^2 + (\cos\theta)^2$

Solution

: $f(\theta) = (\sin\theta)^2 + (\cos\theta)^2 \rightarrow f'(\theta) = 2(\sin\theta)(\cos\theta) + 2(\cos\theta)(-\sin\theta) = 0$ or 0.

Answer : 0

2. Conduct a sequence $\{a_k\}_{k=1}^{\infty}$ with $a_k = r^{1-k}$. Find the value r which satisfies $\sum_{k=1}^{\infty} a_k = 4$

Solution: $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} r^{1-k} = \frac{1}{1-1/r} = \frac{r}{r-1} = 4 \rightarrow r = 4(r-1) = 4r - 4 \rightarrow r = \frac{4}{3}$

Answer : $r = \frac{4}{3}$

3. Suppose that $\vec{a}=(x-4,-1)$ and $\vec{b}=(x,-4)$ are vertical. Find x.

Solution: The inner product $\vec{a}\cdot\vec{b}$ must be zero, given the angle of two vectors is 90 degree (so-called orthogonal). $\vec{a}\cdot\vec{b}=x\times(x-4)+1\times 4=x^2-4x+4=0$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{(-4)^2-4\times1\times4}}{2\times1}=\frac{4\pm\sqrt{0}}{2\times1}=2$

Answer : $x=2$

4. There are 6 male and 5 female students in the program. A group consisting of 3 male and 2 female students will be formed to work on a group project. Find how many different groups can be formed.

Solution: 6C3× 5C2 $=\frac{6\times5\times4}{3\times2\times1}\times\frac{5\times4}{2\times1}=20\times10=200$

Answer : 200