

The Project for Human Resource Development Scholarship
by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test
2017

Solution

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are ‘Basic Math,’ and Part III and Part IV are ‘Applied Math.’ The test result is only for the reference purpose and basically does not affect the selection procedure. However, some accepting universities may require the candidates who apply for the economics-related fields of study to have analytical and numerical skills.

Name : _____

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

$$\begin{aligned} 1. \quad & 1 - (3 - (2 - 5 + 1)) \\ & = 1 - (3 - (-2)) = 1 - 5 = -4 \end{aligned}$$

Answer : -4

$$\begin{aligned} 2. \quad & \left(\frac{1}{3} - \frac{2}{5} \div \frac{3}{10}\right) + \frac{2}{5} \times \frac{15}{4} \\ & = \left(\frac{1}{3} - \frac{4}{3}\right) + \frac{3}{2} = -1 + \frac{3}{2} = \frac{1}{2} \end{aligned}$$

Answer : $\frac{1}{2}$

$$\begin{aligned} 3. \quad & (\sqrt{12} - \sqrt{3}) \times \sqrt{27} \\ & = (2\sqrt{3} - \sqrt{3}) \times 3\sqrt{3} = \sqrt{3} \times 3\sqrt{3} = 3 \times 3 = 9 \end{aligned}$$

Answer : 9

$$\begin{aligned} 4. \quad & \left(3^{1.5} \times \left(\frac{1}{3}\right)^2\right)^4 \times \left(\frac{1}{9}\right)^{-2} \\ & = \left(\left(\frac{1}{3}\right)^{0.5}\right)^4 \times \left(\frac{1}{9}\right)^{-2} = \frac{1}{9} \times \left(\frac{1}{9}\right)^{-2} = 9 \end{aligned}$$

Answer : 9

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equation for x.

$$x + 2 = 1 - \frac{x}{2}$$

$$2x + 4 = 2 - x \rightarrow 3x = -2 \rightarrow x = -\frac{2}{3}$$

Answer : $x = -\frac{2}{3}$

2. Solve the following simultaneous equations for x and y.

$$x + 2y = 8$$

$$x + y = 4$$

$$x = 0, y = 4$$

Answer : $x = 0, y = 4$

3. Consider the straight line in the (x,y)-plane that passes through the points (3,1) and (1,3). Find the slope of the line.

The straight line is described by $y = -x + 4$.

Thus, the slope is -1 .

Answer : -1

4. Find the following sum.

$$\sum_{n=1}^m (2n - 1)$$

$$\sum_{n=1}^m (2n - 1) = 2 \frac{m(m+1)}{2} - m = m(m+1) - m = m^2.$$

Answer : m^2

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Solve the following for x .

$$x^2 = 9$$

$$x = 3, -3$$

$$\underline{\text{Answer : } \quad x = 3, -3}$$

2. Assume that $x < 0$. Find the region of x satisfying the following inequality.

$$\log_{10}|x| < 1$$

Since $x < 0$, we obtain $\log_{10}(-x) < \log_{10}10 \rightarrow -10 < x < 0$

$$\underline{\text{Answer : } \quad -10 < x < 0}$$

3. Find the region of x satisfying the following inequality.

$$\left(\frac{1}{10}\right)^{2x-1} < \left(\frac{1}{10}\right)^3$$

$$2x - 1 > 3 \rightarrow x > 2$$

$$\underline{\text{Answer : } \quad x > 2}$$

4. Suppose that the average of the following five values,
 $\{-2, 5, x - 2, x, 4\}$

is $x + 2$. Derive the value of x .

$$\frac{-2+5+(x-2)+x+4}{5} = x + 2 \rightarrow \frac{5+2x}{5} = x + 2 \rightarrow \frac{2x}{5} = x + 1 \rightarrow 2x = 5x + 5 \rightarrow x = -\frac{5}{3}$$

$$\underline{\text{Answer : } \quad x = -\frac{5}{3}}$$

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1. Determine the second-order derivative of the following. [Notes: e is a mathematical constant which is the base of the natural logarithm. The value of e is approximately equal to 2.71828.]

$$y = 5 - x^2 + e^x$$

$$y' = -2x + e^x \rightarrow y'' = -2 + e^x$$

Answer : $y'' = -2 + e^x$

2. Find the following definite integral.

$$\int_{-1}^0 x^2 dx$$

$$\int_{-1}^0 x^2 dx = \frac{x^3}{3} \Big|_{-1}^0 = \frac{1}{3}$$

Answer : $\frac{1}{3}$

3. Let $A = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$. Find the inverse matrix of A.

$$\det(A) = 4 - 3 = 1 \rightarrow A^{-1} = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$$

Answer : $\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$

4. The profit π is described by the following function:

$$\pi(q) = (100 - 2q)q - 3q^2$$

where q is output. Find the output q at which the profit is maximized.

The first-order condition is: $\pi' = 100 - 4q - 6q = 100 - 10q = 0 \rightarrow q = 10$

Answer : $q = 10$

[PART V] Fill in the following blanks with correct answers.

1. Find the first derivative of the following.

$$f(\theta) = \sin\theta - 2\cos\theta$$

Solution

$$f'(\theta) = \cos\theta - 2(-\sin\theta) = \cos\theta + 2\sin\theta$$

Answer : $f'(\theta) = \cos\theta + 2\sin\theta$

2. Consider a sequence $\{a_k\}_{k=1}^{\infty}$ with $a_k = r^k$. We assume that $0 < r < 1$. Find the value r which satisfies

$$\sum_{k=1}^{\infty} a_k = 2$$

Solution

$$\text{We have } \sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} r^k = \frac{r}{1-r} = 2 \rightarrow r = 2 - 2r \rightarrow r = \frac{2}{3}$$

Answer : $r = \frac{2}{3}$

3. Suppose that $\vec{a} = (1 - x, 2)$ and $\vec{b} = (2, y)$ are vertical and that $x + y = 1$. Find x and y .

Solution

The inner product of $\vec{a} = (1 - x, 2)$ and $\vec{b} = (2, y)$ must be zero.

$$\vec{a} \cdot \vec{b} = 2(1 - x) + 2y = 0 \rightarrow 1 - x + y = 0$$

In addition, since $x + y = 1$, we have $x = 1$ and $y = 0$

Answer : $x = 1, y = 0$

4. There are 5 boys and 4 girls in the class. A team consisting of 2 boys and 3 girls will be formed to work on a group project. Find how many different teams can be formed.

Solution

$${}_5C_2 \times {}_4C_3 = \frac{5 \times 4}{2 \times 1} \times \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 10 \times 4 = 40$$

Answer : 40