

The Project for Human Resource Development Scholarship  
by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test  
2018

Solution

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are ‘Basic Math,’ and Part III and Part IV are ‘Applied Math.’
- The test result is for the reference purpose.

*Name :* \_\_\_\_\_

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

1.  $5 + 2 \times (3 - (1 - 2))$

$$= 5 + 2 \times (3 - (-1)) = 5 + 8 = 13$$

Answer : 13

2.  $\left(1 + \frac{1}{3} \times \frac{4}{5} \div \frac{2}{15}\right) + \frac{5}{2}$

$$= \left(1 + \frac{1}{3} \times \frac{4}{5} \times \frac{15}{2}\right) + \frac{5}{2} = 3 + \frac{5}{2} = \frac{11}{2}$$

Answer :  $\frac{11}{2}$

3.  $(\sqrt{2} - \sqrt{12}) \times (\sqrt{2} + \sqrt{12})$

$$= 2 - 12 = -10$$

Answer : -10

4.  $\left(\left(\frac{1}{2}\right)^{-2} \times \left(\frac{1}{16}\right)^{0.25}\right)^{-2} \div \left(\frac{1}{9}\right)^{-1}$

$$= \left(\left(\frac{1}{2}\right)^{-2} \times \left(\frac{1}{2}\right)^1\right)^{-2} \div \left(\frac{1}{9}\right)^{-1} = \left(\frac{1}{2}\right)^2 \times \frac{1}{9} = \frac{1}{36}$$

Answer :  $\frac{1}{36}$

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equation for  $z$ .

$$\frac{15 - z}{2} = 2z$$

$$4z = 15 - z \rightarrow 5z = 15 \rightarrow z = 3$$

Answer :  $z = 3$

2. Solve the following simultaneous equations for  $x$  and  $y$ .

$$-0.5x + 3y = 9.5$$

$$-x + 2y = 7$$

$$x = -1, y = 3$$

Answer :  $x = -1, y = 3$

3. Consider the straight line in the  $(x,y)$ -plane that passes through the points  $(1,2)$  and  $(a,3)$ . In addition, the slope of the straight line is 3. Find the value of  $a$ .

The straight line is described by  $y = 3x - 1$ . Since the line passes through  $(a,3)$ , it must hold that  $3 = 3a - 1$ . Thus,  $a = \frac{4}{3}$ .

Answer :  $a = \frac{4}{3}$

4. Calculate the following.

$$\sum_{m=1}^{100} (m - 1)$$

$$\sum_{m=1}^{100} (m - 1) = \frac{100(101)}{2} - 100 = 5050 - 100 = 4950.$$

Answer : 4950

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Solve the following for x.

$$4x^2 - 3x = 0$$

$$x = 0, \frac{3}{4}$$

*Answer :*  $x = 0, \frac{3}{4}$

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2. Find the region of x satisfying the following inequality.

$$\log_{10}x < 3$$

Since  $\log_{10}x < 3 = \log_{10}1000$ , we obtain  $0 < x < 1000$

*Answer :*  $0 < x < 1000$

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3. Find the minimum integer x satisfying the following inequality.

$$\sum_{k=1}^x \left(\frac{1}{2}\right)^{1-k} > 60$$

$$2^x - 1 > 60 \rightarrow 2^x > 61 \rightarrow x = 6$$

*Answer :*  $x = 6$

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4. Consider the following five values,

$$\{-1, 4, 2x, -x, 5\}.$$

Suppose that the average of these five values is  $2x$ . Find the median.

$$\frac{-1+4+2x-x+5}{5} = 2x \rightarrow \frac{x+8}{5} = 2x \rightarrow x+8 = 10x \rightarrow 9x = 8 \rightarrow x = \frac{8}{9} \rightarrow$$
$$\{-1, 4, \frac{16}{9}, -\frac{8}{9}, 5\} \rightarrow \text{median is } \frac{16}{9}.$$

*Answer :*  $\text{median} = \frac{16}{9}$

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(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1. Determine the first-order derivative of the following.

$$y = \sqrt{2x}$$

$$y' = \frac{1}{\sqrt{2x}} = \sqrt{\frac{1}{2x}} = (2x)^{-\frac{1}{2}}$$

Answer :  $y' = (2x)^{-\frac{1}{2}}$

2. Solve the following equation for z.

$$\int_{-5}^z x dx = 0$$

Function  $f(x)=x$  is odd function. Thus,  $z = \pm 5$ .

Answer :  $Z = \pm 5$

3. Let  $A = \begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$ . Suppose that the determinant of  $A - B$  is zero. Find the value of  $\lambda$ .

$$\det(A - B) = (1 - \lambda)^2 - 16 = 0 \rightarrow \lambda = -3, 5$$

Answer :  $\lambda = -3, 5$

4. Consider the following function,  $C(x) = x \log_e x$ , where  $x > 0$ . Find  $x$  at which the value of the function is minimized. Note that  $e$  is a mathematical constant which is the base of the natural logarithm

$$\text{The first-order condition is: } C' = \log_e x + 1 = 0 \rightarrow x = e^{-1} = \frac{1}{e}$$

Answer :  $x = \frac{1}{e}$

**[PART V] Fill in the following blanks with correct answers.**

1. Find the first derivative of the following.

$$f(\theta) = (\sin\theta)^2.$$

**Solution**

$$f'(\theta) = 2\sin\theta\cos\theta.$$

*Answer :*  $f'(\theta) = 2\sin\theta\cos\theta.$

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2. Consider a sequence  $\{a_k\}_{k=1}^{\infty}$  with  $a_k = (2r)^{k-1}$ . Find the value  $r$  which satisfies

$$\sum_{k=1}^{\infty} a_k = 10$$

**Solution**

$$\text{We have } \sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} (2r)^{k-1} = \frac{1}{1-2r} = 10 \rightarrow r = \frac{9}{20}$$

*Answer :*  $r = \frac{9}{20}$

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3. Suppose that  $\vec{a} = (3 - x, 1)$  and  $\vec{b} = (x, -2)$  are vertical. Find  $x$ .

**Solution**

The inner product of  $\vec{a} = (3 - x, 1)$  and  $\vec{b} = (x, -2)$  must be zero.

$$\vec{a} \cdot \vec{b} = x(3 - x) - 2 = 0 \rightarrow x^2 - 3x + 2 = 0.$$

Thus,  $x = 1, 2$ .

Answer :  $x = 1, 2$

4. On a circle, 8 points are selected. How many triangles with edges in these points exist?

**Solution**

$${}_8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56.$$

Answer :  $56$