

Japanese Grant Aid for Human Resource Development  
Scholarship (JDS)

Basic Mathematics Aptitude Test  
2015

Solution

Prepared by Graduate School of International Relations, International University of Japan

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are ‘Basic Math,’ and Part III, Part IV, and Part V are ‘Applied Math.’ The test result is only for the reference purpose and basically does not affect the selection procedure. However, some accepting universities may require the candidates who apply for the economics-related fields of study to have analytical and numerical skills.

*Name :* \_\_\_\_\_

(Please show all your work here and write your answers in the designated space)

**[PART I]** Calculate the followings.

1.  $-(2-4)-(3-5)\times 2$

$$= 2 + 4 = 6$$

*Answer :* 6

2.  $\frac{2}{3} - \left\{ 1 - \left( \frac{1}{4} - \frac{1}{2} \right) \div \frac{3}{8} \right\}$

$$= \frac{2}{3} - 1 - \frac{1}{4} \times \frac{8}{3} = -\frac{1}{3} - \frac{2}{3} = -1$$

*Answer :* -1

3.  $\sqrt{4} - \sqrt{\frac{1}{4}}$

$$= 2 - \frac{1}{2} = \frac{3}{2}$$

*Answer :*  $\frac{3}{2}$

4.  $\left( \left( \frac{1}{4} \right)^2 \times 4^{-3} \right)^{\frac{2}{5}} \times \left( \frac{1}{4} \right)^{-2}$

$$= (4^{-5})^{2/5} \times 4^2 = 4^{-2} \times 4^2 = 1$$

*Answer :* 1

(Please show all your work here and write your answers in the designated space)

**[PART II]** Answer the following questions.

1. Solve the following equation for  $z$ .

$$-\frac{1-z}{3} = \frac{z}{2} - 1$$

$$-2 + 2z = 3z - 6 \rightarrow z = 4$$

*Answer:*                       $z = 4$

2. Solve the following simultaneous equations for  $x$  and  $y$ .

$$2x - 5y = -1$$

$$-3x + 7y = 1$$

$$x = 2, y = 1$$

*Answer:*                       $x = 2, y = 1$

3. Consider the straight line in the  $(x,y)$ -plane. Find the slope of the line passing through  $(2,0)$  and  $(0,4)$ .

The straight line is described by  $y = -2x + 4$ .

Thus, the slope is  $-2$ .

*Answer :*                       $-2$

4. Find the following sum.

$$\sum_{k=1}^n 2k$$

$$\sum_{k=1}^n 2k = 2 \frac{n(n+1)}{2} = n(n+1).$$

*Answer :*                       $n(n+1)$

(Please show all your work here and write your answers in the designated space)

**[PART III]** Answer the following questions:

1. Solve the following for  $x$ .

$$x^2 - 5x = -6$$

$$(x - 2)(x - 3) = 0 \rightarrow x = 2, 3$$

*Answer.*                    $x = 2, 3$

2. Find the region of  $x$  satisfying the following inequality.

$$|x - 1| < 1$$

Suppose  $x > 1$ . Then we obtain  $1 < x < 2$ .

Suppose  $x \leq 1$ . Then we obtain  $0 < x \leq 1$ .

Thus,  $0 < x < 2$ .

*Answer.*                    $0 < x < 2$

3. Find the region of  $x$  satisfying the following inequality.

$$\log_{0.1} x < \log_{0.1} 1$$

Since  $\log_{0.1} x$  is decreasing in  $x$ , we obtain  $x > 1$ .

*Answer :*                    $x > 1$

4. Consider the function  $f(x) = 2x - 1$ . Find the inverse of  $f(x)$ . [It is standard to use the notation  $f^{-1}(x)$ .]

$$y = 2x - 1 \rightarrow x = \frac{1}{2}y + \frac{1}{2} \rightarrow y = \frac{1}{2}x + \frac{1}{2} \rightarrow f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$$

*Answer.*                    $\frac{1}{2}x + \frac{1}{2}$

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1. Determine the first-order derivative of the following.

$$y = 2x - x^2$$

$$y' = 2 - 2x.$$

*Answer:* 2 - 2x

2. Find the following definite integral.

$$\int_{-5}^5 x^3 dx$$
$$\int_{-5}^5 x^3 dx = \left. \frac{x^4}{4} \right|_{-5}^5 = 0.$$

*Answer:* 0

3. Let  $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ . Suppose  $A - xI$  is not invertible, so that the determinant of  $A - xI$  is zero. Solve for  $x$ . [Note:  $A$  and  $I$  are  $2 \times 2$  matrix, and  $x$  is a real number.]

$$\det(A - xI) = (1 - x)(-x) - 2 = x^2 - x - 2 = (x - 2)(x + 1) = 0 \rightarrow x = -1, 2.$$

*Answer:*  $x = -1, 2$

4. Find the following value. [Notes:  $e$  is a mathematical constant which is the base of the natural logarithm. The value of  $e$  is approximately equal to 2.71828.]

$$\lim_{x \rightarrow \infty} \left( \frac{e^x}{1 + 5e^x} \right)$$
$$\lim_{x \rightarrow \infty} \left( \frac{e^x}{1 + 5e^x} \right) = \lim_{x \rightarrow \infty} \left( \frac{1}{\frac{1}{e^x} + 5} \right) = \frac{1}{5}.$$

*Answer:*  $\frac{1}{5}$

(Please show all your work here and write your answers in the designated space)

**[PART V]** Fill in the following blanks with correct answers.

1. Assume that  $0 < \theta < \pi/2$ . Solve the following equation for  $\theta$ .  
 $2(\cos\theta)^2 = 3\sin\theta$ .

**Solution**

We have  $2 - 2(\sin\theta)^2 = 3\sin\theta \rightarrow 2(\sin\theta)^2 + 3\sin\theta - 2 = 0$   
 $\rightarrow (2\sin\theta - 1)(\sin\theta + 2) = 0 \rightarrow \sin\theta = 1/2$  (since  $\sin\theta + 2 > 0$ ).  
Since  $0 < \theta < \pi/2$ , we get  $\theta = \pi/6$ .

*Answer :*  $\theta = \pi/6$

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2. Consider a sequence  $\{a_k\}_{k=1}^{\infty}$  with  $a_k = r^{k-1}$ . We assume that  $0 < r < 1$ . Find the value  $r$  which satisfies

$$\sum_{k=1}^{\infty} a_k = 2$$

**Solution**

We have  $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} r^{k-1} = \frac{1}{1-r} = 2 \rightarrow r = \frac{1}{2}$

*Answer :*  $r = \frac{1}{2}$

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3. Suppose that  $\vec{a} = (1 - z, 6)$  and  $\vec{b} = (z, 5)$  are vertical (orthogonal). Find  $z$ .

**Solution**

The inner product of  $\vec{a} = (1 - z, 6)$  and  $\vec{b} = (z, 5)$  must be zero.

$$\vec{a} \cdot \vec{b} = (1 - z)z + 30 = 0 \rightarrow z^2 - z - 30 = (z - 6)(z + 5) = 0 \rightarrow z = -5, 6$$

*Answer :*  $z = -5, 6$

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4. There are 10 boys and 5 girls in the math class. Find the number of ways the instructor can select a team of 3 students from the class to work on a group project. Each team consists of 2 boys and 1 girl.

**Solution**

$${}_{10}C_2 \times {}_5C_1 = \frac{10 \times 9}{2 \times 1} \times \frac{5}{1} = 225.$$

*Answer :*  $225$

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