The Project for Human Resource Development Scholarship by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test 2016

Solution

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are 'Basic Math,' and Part III, Part IV, and Part V are 'Applied Math.' The test result is only for the reference purpose and basically does not affect the selection procedure. However, some accepting universities may require the candidates who apply for the economics-related fields of study to have analytical and numerical skills.

Name : _____

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

1. $3 - 4 \times (1 - 2)$

$= 3 + 4 = 7$

Answer : 7

2. $\left\{\frac{1}{5} \div \left(\frac{2}{5} + \frac{1}{2}\right)\right\} - \frac{1}{3}$

$= \left(\frac{1}{5} \div \frac{9}{10}\right) - \frac{1}{3} = \frac{2}{9} - \frac{1}{3} = -\frac{1}{9}$

Answer : $-\frac{1}{9}$

3. $\sqrt{12} \times \frac{2}{\sqrt{3}}$

$= \sqrt{\frac{12}{3}} \times 2 = \sqrt{4} \times 2 = 2 \times 2 = 4$

Answer : 4

4. $\left\{\left(\frac{1}{27}\right)^{-\frac{1}{3}} \times 27^{\frac{2}{3}}\right\}^{\frac{1}{3}}$

$= \left\{27^{\frac{1}{3}} \times 27^{\frac{2}{3}}\right\}^{\frac{1}{3}} = \left\{27^{\frac{1}{3}+\frac{2}{3}}\right\}^{\frac{1}{3}} = 27^{\frac{1}{3}} = 3$

Answer : 3

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equation for y.

$$3y + 2 = y - \frac{1}{2}$$

➔ $2y = -\frac{5}{2}$ → $y = -\frac{5}{4}$

Answer : $y = -\frac{5}{4}$

2. Solve the following simultaneous equations for x and y.

$$x = y + 2$$
$$2x - 3y = 1$$

➔ $x = 5,\ y = 3$

Answer : $x = 5, y = 3$

3. Consider the straight line in the (x, y)-plane. The straight line passes through $(10,0)$, and the y-intercept of the straight line is $(0,5)$. Find the slope of the straight line.

➔ Since the y-intercept is (0,5), the line is described by $y = ax + 5$. Since the line passes through $(10,0)$, we have $0 = 10a + 5$ or $a = -1/2$.

Answer : slope $= -\frac{1}{2}$

4. Consider the list of numbers, $\{1, 2, 3, \cdots, n\}$. Suppose that the average of n numbers is 100, i.e., $\frac{1}{n}\sum_{k=1}^{n} k = 100$. Find the value of n.

➔ $\frac{\sum_{k=1}^{n} k}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2} = 100$ → $n = 199$

Answer : $n = 199$

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Solve the following for x.

   $2x^2 = (x-1)^2 - 2$

➔ $x^2 + 2x + 1 = 0$ → $(x+1)^2 = 0$ → $x = -1$

Answer : $x = -1$

2. Find the region of x satisfying the following inequality.

   $2|x| < 1$

➔ Suppose $x > 0$. Then we obtain $0 < x < 1/2$. Suppose $x \leq 0$. Then we obtain $-1/2 < x \leq 0$. Thus, $-1/2 < x < 1/2$.

Answer : $-1/2 < x < 1/2$

3. Find the region of x satisfying the following inequality.

   $\frac{1}{125} < \left(\frac{1}{5}\right)^x < 5$

➔ $5^{-3} < 5^{-x} < 5^1$ → $-1 < x < 3$

Answer : $-1 < x < 3$

4. Consider the function $f(x) = \log_{10} x$. Find the inverse function of $f(x)$. [It is standard to use the notation $f^{-1}(x)$.]

➔ Note that the inverse of the log function is the exponential function. Thus, $f^{-1}(x) = 10^x$

Answer $f^{-1}(x) = 10^x$

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1. Find the first-order derivative of the following.

$$y = 1 + \frac{1}{x}$$

➔ $y' = -\frac{1}{x^2}$

Answer : $y' = -\frac{1}{x^2}$

2. Find the following definite integral. [Notes: e is a mathematical constant which is the base of the natural logarithm. The value of e is approximately equal to 2.71828.]

$$\int_0^1 (e^x + 1)dx$$

➔ $\int_0^1 (e^x + 1)dx = e^x + x|_0^1 = (e^1 + 1) - (e^0 + 0) = e.$

Answer : e

3. Let $z = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, and $b = \begin{bmatrix} y \\ 1 \end{bmatrix}$. Find x and y such that $B^{-1}z = b$. [Note: B^{-1} is an inverse matrix of B.]

➔ $B^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ → $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ 1 \end{bmatrix}$
→ $x - y = 2y, x + y = 2$ → $x = \frac{3}{2}, y = \frac{1}{2}$.

Answer : $x = \frac{3}{2}, y = \frac{1}{2}$

4. Consider the profit function, $\pi(q) = (200 - 3q)q - 2q^2$, where q is the output, and $\pi(q)$ is the profit with output q as given. Find the optimal level of output maximizing the profit.

➔ The first-order condition yields $\pi'(q) = 200 - 6q - 4q = 0$ → $q = 20$.

Answer : $q = 20$

(Please show all your work here and write your answers in the designated space)
[PART V] Fill in the following blanks with correct answers.

1. Assume that $0 < \theta < \pi$. Find the region of θ satisfying the following iquality.
   $2(\sin\theta)^2 < 3cos\theta$.

Solution
We have $2 - 2(\cos\theta)^2 - 3\cos\theta < 0 \rightarrow 2(\cos\theta)^2 + 3\cos\theta - 2 > 0$
$\rightarrow (2\cos\theta - 1)(\cos\theta + 2) > 0 \rightarrow \cos\theta > 1/2$ (since $\cos\theta + 2 > 0$).
Since $0 < \theta < \pi$, we get $0 < \theta < \pi/3$.

$Answer:$ $0 < \theta < \pi/3$

2. Consider a sequence $\{c_k\}_{k=1}^{\infty}$ with $c_k = c_0 \times \left(\frac{9}{10}\right)^{k-1}$. Find the value c_0 which satisfies

$$\sum_{k=1}^{\infty} c_k = 100$$

Solution
We have $\sum_{k=1}^{\infty} c_k = c_0 \sum_{k=1}^{\infty} 0.9^{k-1} = \frac{c_0}{1-0.9} = 10c_0 = 100 \rightarrow c_0 = 10$

$Answer:$ $c_0 = 10$

3. Consider two vectors, $\vec{a} = (1, 1)$ and $\vec{b} = (2, 0)$. Let θ denote the angle between $\vec{a}$ and $\vec{b}$. Find the value of $\cos\theta$.

Solution

The inner product of $\vec{a} = (1,1)$ and $\vec{b} = (2,0)$ is $\vec{a} \cdot \vec{b} = 1 \times 2 + 1 \times 0 = 2$. Since $|\vec{a}| = \sqrt{2}$, $|\vec{b}| = 2$, and $\vec{a} \cdot \vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$, we obtain:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Answer : $\cos\theta = \frac{1}{\sqrt{2}}$

4. A committee including 3 boys and 4 girls will be formed from a group of 10 boys and 5 girls. Find how many different committees can be formed from the group.

Solution

${}_{10}C_3 \times {}_5C_4 = \frac{10\times9\times8}{3\times2\times1} \times \frac{5\times4\times3\times2}{4\times3\times2\times1} = 600.$

Answer : 600