

The Project for Human Resource Development Scholarship
by Japanese Grant Aid (JDS)

Basic Mathematics Aptitude Test
2019

Solution

Note:

- You have 60 minutes to complete.
- No calculators are allowed.
- Show all your work and write your answers in the designated space.
- Part I and Part II are 'Basic Math,' and Part III and Part IV are 'Applied Math.'

Name : _____

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

1. $(1 - 3) \times (-5) \times (2 - (1 - 3))$

$$= (-2) \times (-5) \times 4 = 40$$

Answer : 40

2. $\left(\frac{7}{5} \div \frac{2}{15} - \frac{1}{2}\right) \times \frac{1}{3} - \frac{5}{2}$

$$= \left(\frac{7}{5} \times \frac{15}{2} - \frac{1}{2}\right) \times \frac{1}{3} - \frac{5}{2} = \left(\frac{21}{2} - \frac{1}{2}\right) \times \frac{1}{3} - \frac{5}{2} = \frac{10}{3} - \frac{5}{2} = \frac{20 - 15}{6} = \frac{5}{6}$$

Answer : $\frac{5}{6}$

3. $(\sqrt{3} - \sqrt{2})^2 + \sqrt{24}$

$$= 3 - 2\sqrt{6} + 2 + \sqrt{24} = 5 - 2\sqrt{6} + \sqrt{24} = 5 - 2\sqrt{6} + 2\sqrt{6} = 5$$

Answer : 5

4. $\left(\left(\frac{1}{8}\right)^{-1} \div \left(\frac{1}{16}\right)^{0.5}\right)^{-3} \div \left(\frac{1}{3}\right)^2$

$$= (2^3 \div 2^{-2})^{-3} \div \left(\frac{1}{3}\right)^2 = 2^{-15} \times 9 = \frac{9}{32768}$$

Answer : $\frac{9}{32768}$

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equation for w.

$$\frac{5 - w}{2w} = 3$$

$$6w = 5 - w \rightarrow 7w = 5 \rightarrow w = \frac{5}{7}$$

Answer : $w = \frac{5}{7}$

2. Solve the following simultaneous equations for x and y.

$$-x + 3y = 7$$

$$-2x + 2y = 6$$

$$x = -1, y = 2$$

Answer : $x = -1, y = 2$

3. Consider the straight line in the (x,y)-plane that passes through the points (0,a) and (b,0). Assume $2a = 3b$ and $a \neq 0$. Find the slope of the straight line.

The slope of the straight line is described by $-\frac{a}{b} = -\frac{\frac{a}{2a}}{\frac{3}{3}} = -\frac{3}{2}$. Thus, the slope is $-\frac{3}{2}$.

Answer : $-\frac{3}{2}$

4. Solve the following equation for x.

$$|x - 5| = 3$$

$$x - 5 = \pm 3 \rightarrow x = 2, 8$$

Answer : $x = 2, 8$

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Find the region of x satisfying the following inequality.

$$8x^2 - 5x < 9x - 3$$

$$(4x - 1)(2x - 3) = 8x^2 - 14x + 3 < 0 \rightarrow \frac{1}{4} < x < \frac{3}{2}$$

$$\text{Answer : } \frac{1}{4} < x < \frac{3}{2}$$

2. Solve the following equation for x .

$$\log_2(x) = \log_2(6x - 5)$$

$$x = 6x - 5 \rightarrow x = 1$$

$$\text{Answer : } x = 1$$

3. Find the minimum integer x satisfying the following inequality.

$$\sum_{k=1}^x \left(\frac{k}{x}\right) > 100$$

$$\frac{1}{x} \sum_{k=1}^x k > 100 \rightarrow \frac{1}{x} \frac{x(x+1)}{2} > 100 \rightarrow \frac{x+1}{2} > 100 \rightarrow$$

$$x + 1 > 200 \rightarrow x > 199 \rightarrow 200$$

$$\text{Answer : } 200$$

4. Consider the following five values,
 $\{-10, 25, 30, -25, 0\}$.

Suppose that the average of these five values is $\log_3(x)$. Find the value of x .

$$\frac{-10+25+30-25+0}{5} = 4 = \log_3(x) \rightarrow x = 3^4 \rightarrow x = 81$$

$$\text{Answer : } x = 81$$

(Please show all your work here and write your answers in the designated space)

[PART IV] Answer the following questions:

1. Determine the first-order derivative of the following. Note that e is a mathematical constant which is the base of the natural logarithm.

$$y = x^3 + e^{3x} + \log_e x$$

$$y' = 3x^2 + 3e^{3x} + \frac{1}{x}$$

$$\underline{\text{Answer : } y' = 3x^2 + 3e^{3x} + \frac{1}{x}}$$

2. Solve the following equation for z .

$$\int_0^{2z} e^x dx = e^{z+5} - 1$$

$$\int_0^{2z} e^x dx = e^{2z} - 1 = e^{z+5} - 1. \text{ Thus, } e^{2z} = e^{z+5} \rightarrow 2z = z + 5 \rightarrow z = 5.$$

$$\underline{\text{Answer : } z = 5}$$

3. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. Find $A^{-1}B$.

$$\det(A) = 1 - 2 = -1 \rightarrow A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \rightarrow A^{-1}B =$$
$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$$

$$\underline{\text{Answer : } \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}}$$

4. Consider the following function, $F(x, y) = xy$. Assume that $x + y = 10$. Find (x, y) at which the value of the function is maximized.

$$F(x, y) = xy = x(10 - x). \text{ The first-order condition is: } 10 - 2x = 0 \rightarrow$$
$$x = 5, y = 5.$$

$$\underline{\text{Answer : } x = 5, y = 5}$$

[PART V] Fill in the following blanks with correct answers.

1. Find the first derivative of the following.

$$f(x) = 5\sin(5x).$$

Solution

$$f'(x) = 5 \cos(5x) \times 5 = 25 \cos(5x).$$

Answer : 25 cos(5x)

2. Consider a sequence $\{a_k\}_{k=1}^{\infty}$ with $a_k = r^{1-k}$. Find the value r which satisfies

$$\sum_{k=1}^{\infty} a_k = 10$$

Solution

$$\text{We have } \sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} r^{1-k} = \frac{1}{1-1/r} = \frac{r}{r-1} = 10 \rightarrow r = 10r - 10 \rightarrow r = \frac{10}{9}$$

Answer : $r = \frac{10}{9}$

3. Suppose that $\vec{a} = (x, 1)$ and $\vec{b} = (2, y - 2)$ are vertical and that $x + y = 4$. Find x and y .

Solution

The inner product of $\vec{a} = (x, 1)$ and $\vec{b} = (2, y - 2)$ must be zero.

$$\vec{a} \cdot \vec{b} = 2x + y - 2 = 0.$$

Since $x + y = 4$, we have $x = -2, y = 6$.

Answer : $x = -2, y = 6$

4. There are 12 adults and 10 kids in a room. Find the number of ways you can select three persons from them. The choice must contain two kids and one adult.

Solution

$${}_{10}C_2 \times {}_{12}C_1 = \frac{10 \times 9}{2 \times 1} \times 12 = 540.$$

Answer : 540