The Project for Human Resource Development Scholarship (JDS)

Basic Mathematics Aptitude Test 2024

Solution

解答に至るための途中式は黄色のハイライト箇所です。
Part3-2 の問題は途中式が 2 パターンあります。

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings.

1. $2 - (2 - 4 \times (3 + (4 - 7)))$

Solution: $2 - (2 - 4 \times (3 - 3)) = 2 - (2 - 4 \times 0) = 2 - 2 = 0$

Answer : 0

2. $\left(\frac{3}{2} - \frac{1}{2} \div \frac{1}{3}\right) \times \left(\frac{3}{8} \div \frac{3}{16} - \frac{3}{16}\right)$

Solution: $\left(\frac{3}{2} - \frac{1}{2} \div \frac{1}{3}\right) \times \left(\frac{3}{8} \div \frac{3}{16} - \frac{3}{16}\right) = \left(\frac{3}{2} - \frac{1}{2} \times \frac{3}{1}\right) \times \left(\frac{3}{8} \times \frac{16}{3} - \frac{3}{16}\right) = \left(\frac{3}{2} - \frac{3}{2}\right) \times \left(\frac{16}{8} - \frac{3}{16}\right) = 0 \times \frac{29}{16} = 0$

Answer : 0

3. $\left(\sqrt{48}-\sqrt{75}\right)\times\sqrt{3}$

Solution: $\left(\sqrt{48}-\sqrt{75}\right)\times\sqrt{3}==\left(4\sqrt{3}-5\sqrt{3}\right)\times\sqrt{3}=\left(-\sqrt{3}\right)\times\sqrt{3}=-3$

Answer : -3

4. $\left(3^3\times\left(\frac{1}{3}\right)^3\right)^2\div\left(\frac{1}{3}\right)^{-3}$

Solution: $\left(3^3\times\left(\frac{1}{3}\right)^3\right)^2\div\left(\frac{1}{3}\right)^{-3}=(3^3\times3^{-3})^2\div(3^{-1})^{-3}=(3^{3-3})^2\div3^3=1\div3^3=\frac{1}{27}$ or 3^{-3}

Answer : $\frac{1}{27}$ **or** 3^{-3}

(Please show all your work here and write your answers in the designated space)

[PART II] Answer the following questions.

1. Solve the following equations: 4x+2=6x-6

Solution: $4x - 6x = -2 - 6 \rightarrow -2x = -8$, $x = 4$

Answer : 4

2. Solve the following simultaneous equations for x and y.

$3x + 2y - 1 = 7$
$-x + 5y = 3$

Solution: $3x + 2y = 8$, $-x + 5y = 3$ $\rightarrow a: 3x + 2y = 8$, b: $-3x + 15y = 9$
If $a + b = 17y = 17$, y=1. $-x + 5 = 3$ $\rightarrow$ $-x = -2$

Answer: $x = 2, y = 1$

3. Find the region x satisfying the following inequality, where | | indicates the absolute value.

$$\left|\frac{2x+3}{4}\right|<2$$

Solution: $-2 < \frac{2x+3}{4} < 2 \rightarrow -8 < 2x + 3 < 8 \rightarrow -8-3 < 2x < 8 - 3 \rightarrow -11 < 2x < 5$

Answer : $-\frac{11}{2} < x < \frac{5}{2}$ **or** $-5.5 < x < 2.5$ **or** $-5\frac{1}{2} < x < 2\frac{1}{2}$

4. Find the difference between the arithmetic mean and median values in the following observations $\boldsymbol{x_i}$: $Mean(\boldsymbol{x_i}) - Median(\boldsymbol{x_i})$ $where$ $\boldsymbol{x_i} = \{\mathbf{22, 4, 8, 5, 11, 10}\}$.

Solution: Arrange x_i in asceding order as $x_i = \{22,4,8,5,11,10\} = \{4, 5, 8, 10, 11, 22\}$. Given six even elements, the median is the average of the two middle values of 8 and 10 and is equal to 9.

$Mean(x_i) = \frac{4+5+8+10+11+22}{6} = \frac{60}{6} = 10$, $Median(x_i) = \frac{8+10}{2} = 9 \rightarrow Mean(x_i) - Median(x_i) = 10 - 9 = 1$

Answer : 1

(Please show all your work here and write your answers in the designated space)
[PART III] Answer the following questions:

1. Solve the following equation for x. Consider only real number solutions.

$$\frac{5x^3}{2} - 7 = 13$$

Solution: $5x^3 = 20 \times 2 \rightarrow x^3 = 8 = 2^3 \rightarrow \boldsymbol{x = 2}$

Answer： $x = 2$

2. Find the region of x satisfying the following inequality.

$$x^2 - 4x < x - 6$$

Solution: Both factoring and the quadratic formula can be applied to arrive at the solutions:
① $x^2 - 5x + 6 < 0 \rightarrow (x-2)(x-3) < 0 \rightarrow$ Answer: $2 < x < 3$
② $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1} = \frac{5 \pm \sqrt{25-24}}{2 \times 1} = \frac{5 \pm 1}{2 \times 1} = 2,3 \rightarrow$ Answer: $2 < x < 3$

11 月 22 日補記：
以下の回答も正解なのではという問い合わせあり。
試験作成者（片岡先生）へ確認の上、「不等号の問題で範囲を求めているので、正確には正解ではない＝不正解(0 点)」という整理とした。
① (2 ; 3)
② x1=2, x2=3
③ (x-2)(x-3)
④ x1=3, x2=2

Answer： 2 < x < 3

3. Solve the following equation for x.

$$2log_3(x) = log_3(2) + log_3(3x - 4)$$

Solution: $2log_3(x) = log_3(2) + log_3(3x - 4) \rightarrow log_3(x^2) = log_3\big(2 \times (3x - 4)\big) \rightarrow x^2 = 2 \times (3x - 4) \rightarrow x^2 - 6x + 8 = (x - 2) \times (x - 4) = 0$

Answer : x = 2, 4

4. Consider the following six values, [6, 4, 12, 8, 10, 14]. Suppose that the median of six values is 3^{2x}. Find the value of x.

Solution: To find the median (M) from a set of values, follow these steps:
Step 1: Arrange in ascending order: 4, 6, 8, 10, 12, 14
Step 2: Given 6 even elements, the median (M) is the average of the two middle values of 8 and 10.
Median (M) = (8 + 10) / 2 = 9
$9 = 3^{2x} = 3^2 \rightarrow 2x = 2, \; x = 1$

Answer : $x = 1$

(Please show all your work here and write your answers in the designated space)
[PART Ⅳ] Answer the following questions:

1. Determine the first-order derivative of the following. Note that log is the natural logarithm.

$y = log(x^2 + 4)$

Solution: We use the chain rule, which states that if you have a composite function y=f(g(x)), In this case,
f= $log(\cdot)$ and g= $(x^2 + 4)$ $y' = f'\big(g(x)\big) \cdot g'(x) = \frac{1}{(x^2+4)} \cdot 2x = \frac{2x}{(x^2+4)}$

Answer : $\frac{2x}{(x^2 + 4)}$

2. Find the following definite integral.

$\int_1^2 (\mathbf{2x} + \mathbf{3}x^2)dx$

Solution: $\int_1^2 (2\text{x} + 3x^2)dx = \left(2 \times \frac{1}{2}x^2\right) + \left(3 \times \frac{1}{3}x^3\right)\Big]_1^2 = (4 + 8) - (1 + 1) = 10$

Answer : 10

3. Let A=$\begin{bmatrix} 1 & 1 \\ -2 & a \end{bmatrix}$ and B=$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$.
Assume determinant of the matrix A is 5. Find $A^{-1} \times B$.

Solution: The determinant of the matrix, denoted as $\det(A) = 1 \times a - (-2 \times -1) = 1.$ → a=3

$A^{-1} = \frac{1}{5} \times \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.2 \\ 0.4 & 0.2 \end{bmatrix}$. $A^{-1}B = \begin{bmatrix} 0.6 & -0.2 \\ 0.4 & 0.2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.2 \\ 0.8 & 0.8 \end{bmatrix}$

Answer : $\begin{bmatrix} 0.2 & 0.2 \\ 0.8 & 0.8 \end{bmatrix}$ ***or*** $\begin{bmatrix} 1/5 & 1/5 \\ 4/5 & 4/5 \end{bmatrix}$

4. Find the maximum total revenue (TR) for a firm, given the flowing functions: $TR = P \times Q$ and $P = 16 - 2Q$, where P and Q are the prince and quantity of goods, respectively.

Solution: $\text{TR} = P \times Q = (16 - 2Q) \times Q = 16Q - 2Q^2$.

To find the maximum value, the first derivative of TR with respect to Q is equal to zero. In Maximization $\text{TR}' = -4\text{Q} + 16 = 0 \rightarrow Q = 4.$ Thus, the maximum TR is $Max(\text{TR}) = 16Q - 2Q^2 = 16 \times 4 - 2 \times 4^2 = 32$ where P=8.

Answer : 32

(Please show all your work here and write your answers in the designated space)
[PART V] Answer the following questions:

1. Find the following trigonometric function value: $\boldsymbol{sin}\left(\pi - \frac{5\pi}{6}\right) + cos\left(\frac{\pi}{3}\right)$ where π represents the mathematical constant and the angles are in radians.

Solution: At first , we simplify the expression inside the sine function of the first term.$\pi - \frac{5\pi}{6} = \frac{\pi}{6}$. $sin\left(\frac{\pi}{6}\right) + cos\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{1}{2} = 1$

Answer : 1

2. Given a sequence $\left(\frac{1}{3}\right)^{\boldsymbol{n-1}}$, find $\sum_{n=1}^{\infty} (\boldsymbol{1}/\boldsymbol{3})^{\boldsymbol{n-1}}$ where $\boldsymbol{n}$ is an integer.

Solution: Use $\sum_{n=1}^{\infty} ak^{n-1} = \left(\frac{a}{1-k}\right)$ where a and k are first term of the geometric sequence and common ratio, respectively. Substituting a=1 and $k = \left(\frac{1}{3}\right)$, we have $\frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

Answer : $\frac{3}{2}$ ***or* 1.5 or** $1\frac{1}{2}$

3. Suppose that $\vec{a}= (4x-5, 2)$ and $\vec{b}= (3, y)$ are vertical and that x − y=3. Find x and y.

Solution: The inner product $\vec{a}\cdot\vec{b}$ must be zero, given the angle of two vectors is 90 degree (so-called orthogonal). $\vec{a}\cdot\vec{b}= 3\times(4x-5)+2y = 12x+2y-15 = 0$So, we solve the simultaneous equations of $12x+2y-15=0$ and $x-y=3$.
Thus, $x=\frac{3}{2}$, y=$-\frac{3}{2}$.

Answer : $x=\frac{3}{2}$, y=$-\frac{3}{2}$ ***or*** $x=1\frac{1}{2}$, y=$-1\frac{1}{2}$ **or** $x=1.5$,y=-1.5

4. In the Econometrics course, there are 5 Japanese students and 7 non-Japanese students. The professor plans to form a group for assignment, by selecting two members from each group. Find the total number of different teams that can be formed.

Solution: ${}_5C_2\times{}_7C_2 = \frac{5\times4}{2\times1}\times\frac{7\times6}{2\times1} = 10\times21 = 210$

Answer : 210