The Project for Human Resource Development Scholarship (JDS)

Basic Mathematics Aptitude Test 2025

Solution

解答に至るための途中式は黄色のハイライト箇所です。
PART III の問題２は途中式が２パターンあります。

(Please show all your work here and write your answers in the designated space)
[PART I] Calculate the followings.

1. $5 - 2 \times [3 + (6 - 9)]$

Solution: $5 - 2 \times [3 + (6 - 9)] = 5 - 2 \times [3 - 3] = 5 - 0 =$ **5**

Answer: 5

2. $\left(\frac{3}{4} \div 1.5\right) \div \left(\frac{3}{10} - \frac{4}{5}\right) + \frac{3}{5} \div \left(\frac{2}{5} - 0.2\right)$

Solution: $\left(\frac{3}{4} \div 1.5\right) \div \left(\frac{3}{10} - \frac{4}{5}\right) + \frac{3}{5} \div \left(\frac{2}{5} - 0.2\right) = \left(\frac{3}{4} \div \frac{3}{2}\right) \div \left(\frac{3}{10} - \frac{4}{5}\right) + \frac{3}{5} \div \left(\frac{2}{5} - 0.2\right)$
$= \left(\frac{3}{4} \times \frac{2}{3}\right) \div \left(\frac{3}{10} - \frac{8}{10}\right) + \frac{3}{5} \div \left(\frac{2}{5} - \frac{1}{5}\right) = \frac{1}{2} \div \left(-\frac{1}{2}\right) + \frac{3}{5} \div \frac{1}{5} = \frac{1}{2} \times (-2) + \frac{3}{5} \times 5 = -1 +$
$3 = 2$

Answer: 2

3. $\left(\sqrt{12}-\sqrt{27}\right)+\sqrt{3}$

Solution: $\left(\sqrt{\mathbf{12}}-\sqrt{\mathbf{27}}\right)+\sqrt{\mathbf{3}}=\left(\mathbf{2}\sqrt{\mathbf{3}}-\mathbf{3}\sqrt{\mathbf{3}}\right)+\sqrt{\mathbf{3}}=\left(-\sqrt{\mathbf{3}}\right)+\sqrt{\mathbf{3}}=\mathbf{0}$

Answer: 0

4. $\left(3^2\times\left(\frac{1}{3}\right)^2\right)^3\div\left(\frac{1}{3}\right)^2$

Solution: $\left(3^2\times\left(\frac{1}{3}\right)^2\right)^3\div\left(\frac{1}{3}\right)^2=\left(\mathbf{3^2\times 3^{-2}}\right)^{\mathbf{3}}\div\left(\mathbf{3^{-1}}\right)^{\mathbf{2}}=\left(\mathbf{3^{2-2}}\right)^{\mathbf{3}}\times\mathbf{3^2}=\mathbf{1}\times\mathbf{3^2}=\mathbf{9}$

Answer: 9

(Please show all your work here and write your answers in the designated space)
[PART II] Answer the following questions.

1. Solve the following equations: $4x - 2 = 2x - 8$

Solution: $4x - 2 = 2x - 8 \rightarrow 4x - 2x = -8 + 2 \rightarrow \mathbf{2x = -6}$, $\mathbf{x = -3}$

Answer: $\mathbf{-3}$

2. Solve the following simultaneous equations for x and y.

$$4x + 6y - 4 = 10$$
$$-2x + 8y = 4$$

Solution: $\mathbf{2} \times (2x + 3y - 2 = 5) \rightarrow 2x + 3y - 2 + -2x + 8y = 4 + 5 \rightarrow 11y = 11, \rightarrow y = 1$, x=2

Answer: $\boldsymbol{x = 2, y = 1}$

3. Find the region x satisfying the following inequality, where | | indicates the absolute value.

$|x+1| \leq 2$

Solution: $-2 \leq x+1 \leq 2 \rightarrow -2-1 \leq x \leq 2-1 \rightarrow -3 \leq x \leq 1$

Answer: $-3 \leq x \leq 1$

4. Find the arithmetic mean difference between the following observations of two variables $\boldsymbol{x_i}$ **and** $\boldsymbol{y_i}$, **denoted as** $Mean(\boldsymbol{x_i}) - Mean(\boldsymbol{y_i})$, where $\boldsymbol{x_i} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}\}$ **and** where $y_i = \{\mathbf{2}, \mathbf{4}, \mathbf{6}, \mathbf{8}, \mathbf{10}\}$

Solution: $Mean(\boldsymbol{x_i}) = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3,\ Mean(\boldsymbol{y_i}) = \frac{2+4+6+8+10}{5} = 6$
$\rightarrow Mean(\boldsymbol{x_i}) - Mean(\boldsymbol{y_i}) = 3 - 6 = -3$

Answer: -3

(Please show all your work here and write your answers in the designated space)

[PART III] Answer the following questions:

1. Solve the following equation for x. Consider only real number solutions.

$$\frac{x^2}{3} - 4 = 8$$

Solution: x^2 =12 $\times$ 3 $\rightarrow$ x^2 =$(\pm 6)^2$

Answer: x= 6, -6 or ± 6

2. Find the region of x satisfying the following inequality.

$$3x^2 - 5x < x^2 + 2x - 3$$

Solution: Both factoring and the quadratic formula can be applied to arrive at the solutions:

$\mathbf{2x^2 - 7x + 3 < 0 \rightarrow (2x-1)(x-3) < 0}$ **→Answer:** $\mathbf{\frac{1}{2} < x < 3}$

$\mathbf{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2} = \frac{7 \pm \sqrt{49-24}}{4} = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4} = \frac{1}{2}, 3}$ **→Answer:** $\mathbf{\frac{1}{2} < x < 3}$

Answer: $\mathbf{\frac{1}{2} < x < 3}$ **or 0.5<** $\mathbf{x < 3}$

3. Solve the following equation for x.

$$2log_3(x) = log_3(4 + 2x - \boldsymbol{x^2})$$

Solution: $\boldsymbol{2log_3(x) = log_3(4 + 2x - x^2) \rightarrow 2x^2 - 2x - 4 = (2x - 4)(x + 1) = 0\ Since\ x > 0, x = 2.}$

Answer: $\boldsymbol{x = 2}$

4. Consider the following six values, [8, 5, 4, 10, 14, 12]. Find $log_3(x)$ where x is the median of the six values.

Solution: To find the median (M) from a set of values, follow these steps:

Step 1: Arrange in ascending order: 4, 5, 8, 10, 12, 14

Step 2: Given 6 even elements, the median (M) is the average of the two middle values of 8 and 10.

Median (M) = (8 + 10) / 2 = 18 / 2 = 9
$x = 9 \rightarrow \log_3(9) = \log_3 3^2 = 2$

Answer: $log_3 x = \boldsymbol{2}$

(Please show all your work here and write your answers in the designated space)
[PART IV] Answer the following questions:

1. Determine the first-order derivative of the following. Note that log is the natural logarithm.

$y = log_e x + 3x^0$

Solution: $y = log_e x + 3x^0 = log_e x + 3 \rightarrow \boldsymbol{y' = \frac{1}{x}}$

Answer: $\boldsymbol{y' = \frac{1}{x}}$

2. Find the following definite integral.

$\int_0^2 (2x + 3x^2)dx$

Solution: $\int_0^2 (2x + 3x^2)dx = \int_0^2 (2\mathrm{x})dx + \int_0^2 (3x^2)dx = \boldsymbol{(x^2)\big]_0^2 + (x^3)\big]_0^2 = (2^2 + 2^3) = 12}$

Answer: **12**

3. Let A=$\begin{bmatrix} a & -1 \\ -2 & 3 \end{bmatrix}$ and B=$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$.

Assume determinant of matrix A is 1. Find $A^{-1}B$.

Solution: The determinant of the matrix, denoted as $\det(A) = a \times 3 - (-2 \times -1) = 1. \rightarrow$ a=1

$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ $\quad A^{-1}B = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Answer: $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

4. Find the value of x and y that solves the following constrained maximization problem:

   Maximize $\sqrt{3xy}$ subject to $3x + y = 12$.

Solution: $3x + y = 12 \rightarrow y = 12 - 3x$.

$\sqrt{3xy} = \sqrt{3x(12 - 3x)} = \sqrt{36x - 9x^2}$ →Deffreciate wrt x: $\frac{df}{dx} = \frac{1}{2\sqrt{36x-9x^2}} \cdot (36 - 18x)$.

To maximize, set $\frac{df}{dx}$ =0. $(36 - 18x) = 0 \quad \rightarrow x$ =2, y=6

Answer: x =2, y=6

(Please show all your work here and write your answers in the designated space)

[PART V] Answer the following questions:

1. Find the first derivative of the following. $f(\theta) = \sin\left(\frac{3}{2}\theta\right)$

Solution: $\boldsymbol{f(\theta) = \sin\left(\frac{3}{2}\theta\right) \quad \rightarrow f'(\theta) = \frac{3}{2}\cos\left(\frac{3}{2}\theta\right)}$

Answer: $\boldsymbol{\frac{3}{2}\cos\left(\frac{3}{2}\theta\right)}$ or 1.5 cos (1.5θ)

2. Given a sequence $\boldsymbol{4^{n-1}}$, find the value $\sum_{n=1}^{3} \boldsymbol{4^{n-1}}$, where $\boldsymbol{n}$ is an integer.

Solution: The n^th term of a geometric series is given as $\boldsymbol{ar^{n-1}}$, where *a* and *r* are first term of the geometric sequence and common ratio, respectively. Using the sum of the first *n* terms of a geometric series is given by the formula. $\boldsymbol{S_n = \frac{a(1-r^n)}{1-r}}$ where r>1.

$\sum_{n=1}^{3} \boldsymbol{4^{n-1} = \frac{1\times(1-4^3)}{1-4} = \frac{1\times(1-64)}{1-4}}$ **=1 + 4 + 16 =21**

Answer: 21

3. Suppose that $\vec{a} = (3x + 6, 3)$ and $\vec{b} = (2, y)$ are vertical and that $x + y = -2$.
Find x and y.

Solution: The inner product $\vec{a}\cdot\vec{b}$ must be zero, given the angle of two vectors is 90 degree (so-called orthogonal). $\vec{a}\cdot\vec{b} = 3 \times (2x + 4) + 3y = 6x + 3y + 12 = 0 \rightarrow 6x + 3y = -12$. So, we solve the simultaneous equations of $6x + 3y = -12$ and $x + y = -2$. Thus, $x = -2$, $y = 0$

Answer: **$x = -2$, $y = 0$**

4. In the public policy course, there are six students majoring in economics and five students majoring in public management. The professor plans to form a group work by selecting three members from each major group. Find the total number of different teams that can be formed.

Solution: ${}_6C_3 \times {}_5C_3 = \frac{6\times5\times4}{3\times2\times1} \times \frac{5\times4\times3}{3\times2\times1} = 20 \times 10 = \mathbf{200}$

Answer: **200**